6/30/2019

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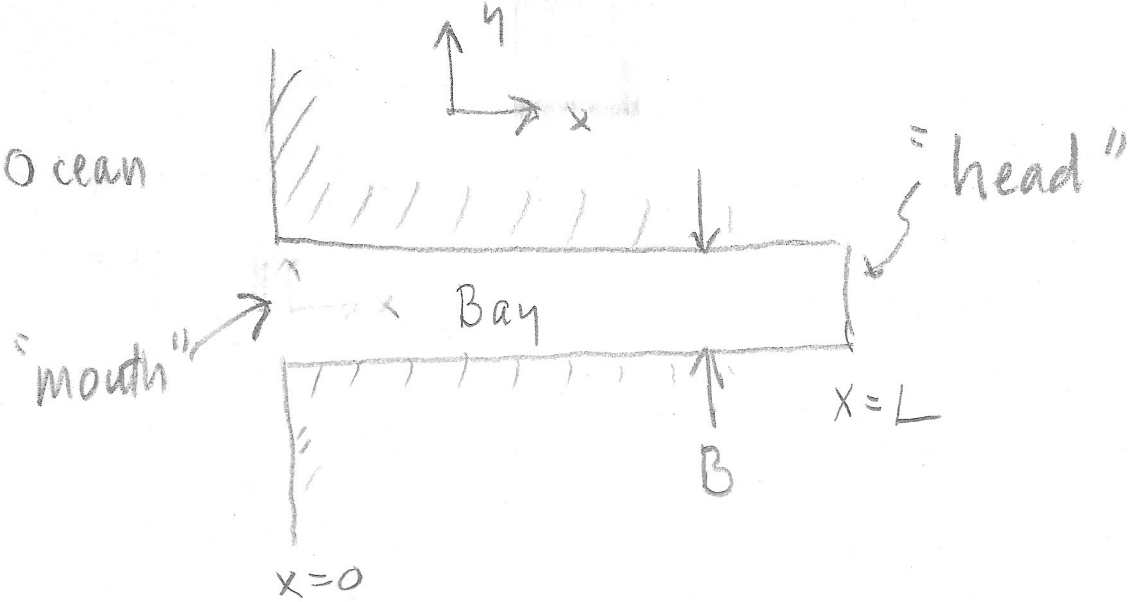
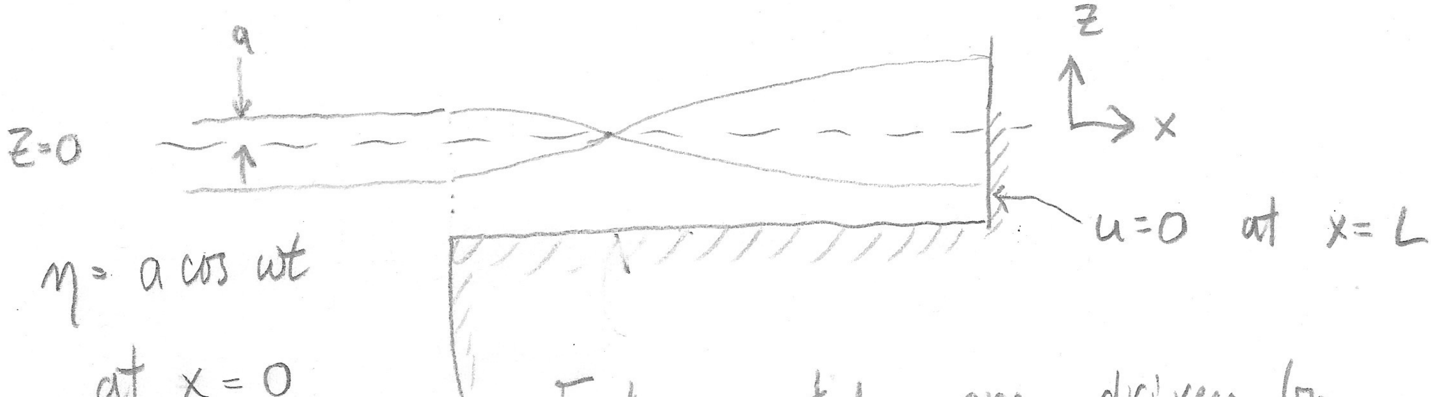
Frictional SW waves in a channel connected to the ocean.





Estuary tides are driven by ocean tides at the mouth.

- Response in the bay depends strongly on friction and resonance.

Linear, frictional SW equations (drop $(\bar{\ })$)

$\boxed{x \text{ mom}}$ $$u_t + g\eta_x + Ru = 0$$ (+)

$\boxed{\text{mass}}$ $$\eta_t + Hu_x = 0$$

- Form wave eqn. with $R=0$ and show $c = \frac{\omega}{k} = \sqrt{gH}$
- Also, for a wave of form $\eta = a\cos(kx - \omega t)$ what is u? Express your answer in terms of a, c, and H.
- What does this tell you about tidal currents in shallow vs. deep water?

General solution to the linear system forced at frequency ω will only respond at that frequency, so guess a form of the solution:

$$u = \mathrm{Re}\{\mathcal{U} \exp(-i\omega t)\}$$
$$\eta = \mathrm{Re}\{E \exp(-i\omega t)\} \qquad (++)$$

where $\mathcal{U}$ and E are unknown complex functions of x only.

- We proceed by working out the full complex solutions and only evaluate the Real part when satisfying the real boundary conditions at the end.

Plugging (++) into (+)

$$-i\omega \mathcal{U} + g E_x + R\mathcal{U} = 0 \qquad (i)$$

$$-i\omega E + H \mathcal{U}_x = 0 \qquad (ii)$$

Eliminate u:

From (ii) $u_x = \frac{i\omega}{H} E$

From $\frac{\partial}{\partial x}$ (i) $-i\omega u_x + g E_{xx} + R u_x = 0$

$$\Rightarrow \quad E_{xx} + \left(\frac{\omega^2}{gH} + \frac{i\omega R}{gH} \right) E = 0$$

or $$\boxed{E_{xx} + k^2 E = 0} \quad (*)$$

where we have defined complex wave # k

$$k = \frac{\omega}{c} \sqrt{1 + \frac{iR}{\omega}} \qquad \text{where } c = \sqrt{gH}$$

and $(*)$ has solutions of the form

$$E = \underset{\substack{\longrightarrow \\ \text{incident}}}{\alpha^+ \exp i(kx)} + \underset{\substack{\longleftarrow \\ \text{reflected}}}{\alpha^- \exp i(-kx)}$$

and $\alpha^{+/-}$ are complex constants we can choose to satisfy the boundary conditions.

Recall: mouth boundary condition

was $\eta = a \cos \omega t$ at $x = 0$.

- Simplest solution: <u>no friction</u>, infinite channel

  $\Rightarrow$ no reflected wave

  so chosing $\alpha^+ = a$ to satisfy the b.c.

$$\eta = \mathrm{Re}\left\{ a \exp i (kx - \omega t) \right\} = a \cos(kx - \omega t)$$

a progressive wave with phase speed

$$c = \frac{\omega}{k} = \sqrt{gH} \quad \checkmark$$

$\boxed{\text{Insert Velocity Lecture}}$

- Solution with a reflected wave, and friction.

  Then we need to satisfy $u(L) = 0$

  From $\boxed{\text{x mom}}$ to have $u = 0 \Rightarrow \eta_x = 0$

  at $x = L$